The Stability Property of Cognitive Radio Systems with Imperfect Sensing

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Abstract

In this paper, we study the stability property of a cognitive radio system comprised of a set of source-destination pairs having different priorities. In particular, we focus attention on the effect of imperfect sensing on the stability region of the system, which has been overlooked in most of related previous work. The adopted cognitive access protocol allows the secondary user not only to exploit the idle slots of the primary user but also to transmit along with the primary user with some probability. This is aimed at achieving the full utilization of the shared channel with capture, i.e., a transmission can be correctly decoded at the destination, even in the presence of other transmissions, if the received signal-to-interference-plus-noise ratio (SINR) exceeds a certain threshold for successful decoding. The abolition of strong primacy, however, requires the secondary user to properly regulate its multi-access probability in order not to impede the primary user’s stability guarantee. To this end, the maximum stability region of the system is characterized which describes the theoretical limit on rates that can be pushed into the system while maintaining the queues stable. Interestingly, we found that even with non-zero sensing error rates, there exists a condition for which we can achieve the identical stability region that is achieved with perfect sensing. This is when the destinations enjoy fairly strong capture, and if then sensing errors do not affect the stability region for the queueing system. For the case when the specified condition does not hold, we precisely quantify the loss due to the imperfect sensing in terms of the size of the stability region. Finally, we study the problem of controlling the operating point of the sensing device over its receiver operating characteristic (ROC) and summarize some key aspects observed in the control.

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Index Terms

Cognitive radio, imperfect channel sensing, queueing stability, capture effect

I. INTRODUCTION

There is an increasing demand for variety of wireless devices and applications for our daily lives, but the usable electromagnetic radio spectrum is of limited physical extend. Recent studies on the spectrum usage have revealed that substantial portion of the licensed spectrum is underutilized, which arouses a pressing need for developing a new technique for improved spectrum utilization [1]. The cognitive radio communications, a means of opening up licensed bands to unlicensed users, have the potential to become a solution to the spectrum underutilization problem [2]–[4]. The high-priority user, often called as the primary (or licensed), is allowed to access the spectrum whenever it needs, while the low-priority user, called as the secondary, is required to make a decision on its transmission based on what the primary user does.

In order to put our contribution in perspective, we start with some background study. In [5], an opportunistic scheduling policy for cognitive access systems was developed. It is based on the collision channel model, in which if more than one users transmit at the same time, none of them are successful. This is too pessimistic in the sense that a transmission may succeed even in the presence of interference, which is called capture effect [6]. In this work, we take into account the realistic effect of wireless signal transmission by incorporating the physical layer parameters, and the transmission by each node is successful if the strength of the received signal relative to the interference is sufficiently high. For such a practical system based on the signal-to-interference-plus-noise-ratio (SINR), applying the average collision rate constraint imposed in [5] is not necessary or even impossible since the interference caused by the secondary transmitter has complex effect on the performance of the primary communication system. In such a context, we consider the primary communication system having bursty packet arrivals and aim at guaranteeing its stability, whereas in [5] it was simplified as two-state ON/OFF Markov process whose activity is not even affected by the collisions. In other words, even if the primary user’s packet is lost due to the collision caused by a secondary user, the primary user does not attempt to retransmit the lost packet. This is suitable for the primary network servicing real-time applications such as the video streaming which do not rely on retransmissions to achieve reliability because they cannot tolerate much delay. In such cases, the secondary network is required to defer its transmissions whenever
there exist primary’s transmissions, or the primary network is expected to impose strong protection on its transmission, i.e., low rate encoding with high redundancy, such that it is guaranteed that primary network transmissions are successful with high probability. In this work, on the other hand, we consider a packet-based general data network, which requires strict reliability guarantee on transmissions. This includes, for example, the primary network servicing best-effort Internet traffic or any kind of applications requiring 100% throughput but rather delay tolerant. Thus, under our model, lost packets must be retransmitted through a medium access control (MAC) protocol such as the automatic-repeat-request (ARQ) and those retransmissions would certainly affect the primary user’s activity.

In [7], the unrealistic assumption made in [5] mentioned above was corrected for a reduced system model consisting of a single primary and secondary user. In such an effort, an active period that the primary user transmits successively until it becomes idle was defined as an interval that the primary user’s packet queue is non-empty. This is based on the assumption that the primary user transmits whenever its queue is non-empty. Furthermore, the effect of interference caused by the secondary user is reflected in the primary user’s activity through the queueing dynamics by lowering the service rate when there is a concurrent transmission. For such a model, a joint flow control and power allocation policy was obtained, but the derivation is based on the assumption that the primary user is always stable. In the absence of the knowledge on the network stability region\(^1\), however, it is infeasible to judge the stability of the primary user’s queue a priori, and the characterization of the stability region is usually not an easy task especially when the network nodes are interacting, i.e., the service process of one depends on the status of the others.

In [8], the interaction between users was fully taken into account for a similar network model with that considered in [7]. In contrast to the traditional notion of cognitive radio, in which the secondary user is required to relinquish the channel as soon as the primary user is detected, the secondary user is allowed to not only exploit the idle slots of the primary user but also to transmit along with the primary user with some probability to attain full utilization of the shared channel with capture. Such abolition of strong primacy, however, requires the secondary user to properly control its multi-access probability in the way that it does not hamper the stability of the primary user at any given input rate whenever it is stabilizable.

\(^1\) Stability region is defined as the set of arrival rate vectors for which the queues in the system are stable, and a queue is said to be stable if it reaches a steady state and does not drift to infinity. A formal definition is given in Section II.
In [9, the approach in [8] was further extended to the scenario when the primary user is powered from a randomly time-varying renewable energy source and has a battery for storing the harvested energy. The limited energy availability imposed by the battery status results in a reduced stability region, which is precisely quantified in the paper.

Most importantly, all the above-mentioned studies, [5], [7–9], were performed based on the ideal assumption that the secondary user always knows the exact activity of the primary user without an error. In reality, however, such knowledge is acquired through a certain decision process at the secondary user, which is subject to the occurrence of errors as long as there exists randomness in the observed signal. In [10, the performance of the energy detector, which is popular due to its generosity and low complexity [11], was derived in terms of the probabilities of false alarm and miss, which are functions of the sensing duration, the sampling rate, and the received signal-to-noise-ratio (SNR). After that, the sensing duration was further optimized in order to maximize the secondary user's throughput at given target error probabilities, but the primary user's activity was abstracted as in [5] as a random process with fixed a priori probability. In [12], the approach in [10] was extended to the multi-channel scenario, and a cooperative sensing scheduling policy was proposed to detect the activities over the channels.

In this paper, we focus attention on the effect of practical spectrum sensing on which the overall performance of the cognitive access system depends. The opportunistic cognitive access protocol proposed in [8] is considered again for the system consisting of a single primary and secondary source-destination pair as shown in Fig. 1. The primary user transmits uninterruptedly whenever its queue is non-empty, which is independent of actions made by the secondary user. On the other hand, the transmission by the
secondary user is chosen in a careful manner that does not hamper the primary user’s stability guarantee. The secondary user first observes the activity of the primary user and, if it is sensed to be idle, the secondary user transmits with probability 1 if its packet queue is non-empty. Otherwise, if the primary user is sensed to be active, the secondary user transmits with some probability $p$ to take advantage of the capture effect although, at the same time, it risks impeding the primary user’s success. Our design objective is, therefore, to optimally choose the multi-access probability $p$ by the secondary user so as to maximize its own stable throughput while ensuring the stability of the primary user at given input rate demand in the presence of sensing errors at the secondary user. This enables us to characterize the maximum achievable stability region of the system at given sensing error rates.

Our contributions in this work can be summarized as follows. First, we introduce a practical model for cognitive access systems. Specifically, when compared to the previous work that oversimplified the primary user’s activity [5], [10], [12], it is precisely modeled through the queueing dynamics which is also subject to the interference caused by the secondary user. Furthermore, the imperfect spectrum sensing, one of the most practical aspects of cognitive access systems, is also incorporated in the model. Secondly, the impact of imperfect sensing on the stability of the cognitive access systems is precisely analyzed. The remarkable result is that there exists a condition for which we can achieve the identical stability region that is achieved with perfect sensing, and the condition is expressed in terms of values of physical layer parameters. This is when the destinations enjoy fairly strong capture, and if then sensing errors do not affect the stability region for the queueing system; for arrival rates within the stability region, the secondary user can transmit whenever it has a packet to send, independent of primary user activity on the channel and without destabilizing the primary user’s queue. For the case when the condition does not hold, we quantify the loss due to the imperfect sensing in terms of the size of the stability region when compared against the case with perfect sensing. Finally, we study the relationship of the specificity of the detector to the size and shape of the stability region, which indicates how one could tune the detector according to desired performance for the network.

The rest of the paper is organized as follows. In Section II, we present the system model and revisit the notion of stability. In Section III, we describe our main result on the stability region of the cognitive access systems in the presence of spectrum sensing errors. The proof of our main result is given in Section IV, which is based on the stochastic dominance technique previously introduced in [13] to deal with interacting queues. Finally, we draw some conclusions in Section VI.
II. SYSTEM MODEL

We consider a system consisting of two source-destination pairs, the primary pair \((s_1, d_1)\) and the secondary pair \((s_2, d_2)\), as shown in Fig. 1. Each source \(s_i, i \in \{1, 2\}\), has an infinite size queue for storing the arriving packets of fixed length. Time is slotted and the slot duration is equal to a packet transmission time. As illustrated in Fig. 2, the primary user’s transmission consists of the preamble symbols followed by the encoded data symbols of a packet, if the primary user transmits during time slot \(n\). Otherwise, if the primary user does not transmit, the entire slot is unused. It is assumed that the secondary user knows the exact timing of the primary user’s frame and performs sensing during the preamble symbol duration. Once the secondary user decides to transmit, it transmits over the primary user’s data symbol duration in a synchronous manner. It is assumed that the acknowledgments (ACKs) on the success of transmissions are sent back from the destinations to the corresponding sources instantaneously and error-free. Note that the feedback on simple ACK/NACK can be expressed with one bit message, and such a message can be reliably transmitted with robust coding through separate control channel even with negligible bandwidth. Even so, delayed, corrupted, or lost ACK messages are expected in practice and will lead to more packet retransmissions, larger queues and, thus, smaller stability region than those predicted in this work obtained under ideal assumption.

Let \(Q_i(n)\) denote the number of packets buffered at \(s_i\) at the beginning of the \(n\)-th slot which evolves according to

\[
Q_i(n + 1) = \max[Q_i(n) - \mu_i(n), 0] + A_i(n)
\]

where the stochastic processes \(\{\mu_i(n)\}_{n=0}^{\infty}\) and \(\{A_i(n)\}_{n=0}^{\infty}\) are sequences of binary random variables representing the number of services and arrivals at \(s_i\) during time slot \(n\), respectively. The arrival process \(\{A_i(n)\}_{n=0}^{\infty}\) is modeled as an independent and identically distributed (i.i.d.) Bernoulli process with \(E[A_i(n)] = \lambda_i\), and the processes at different nodes are assumed to be independent of each other. The
service process \( \{\mu_i(n)\}_{n=0}^{\infty} \) depends jointly on the transmission protocol, sensing errors, and the underlying channel model, which governs the success of transmissions. In the considered cognitive access protocol, \( s_1 \) transmits whenever \( Q_1(n) \neq 0 \). That is, the head-of-the-line (HOL) packet is transmitted from \( s_1 \) whenever the queue at \( s_1 \) is non-empty. The packet remains at the HOL position and retransmitted until an acknowledgement indicating the successful receipt at the destination is received at the source. On the other hand, \( s_2 \) adapts its transmission based on the observation made on the activity of \( s_1 \). Given that \( Q_2(n) \neq 0 \), \( s_2 \) transmits with probability 1 if \( s_1 \) is observed to be idle. Although \( s_1 \) is observed to be active, \( s_2 \) transmits with probability \( p \) to take advantage of the capture. Note that the lost packet from \( s_2 \) is also retransmitted until it is successfully received at the corresponding destination.

Note that \( s_1 \) can be falsely perceived to be active by \( s_2 \) when indeed it is idle or falsely perceived to be idle when it is active, which are called false alarm and miss, and their rates are denoted by \( \epsilon_f \) and \( \epsilon_m \), respectively. The channel model used in this work is a generalized form of the packet-erasure model, which well reflects the effect of fading, attenuation, and interference at the physical layer [14]–[17]. Denote with \( q_{i|M} \) the success probability of user \( s_i \) when a set \( M \) of users are transmitting simultaneously. It is related to the physical layer parameters through

\[
q_{i|M} = \Pr[\gamma_{i|M} \geq \theta]
\]

where \( \gamma_{i|M} \) denotes the signal-to-interference-plus-noise-ratio (SINR) of the signal transmitted from \( s_i \) at the designated destination \( d_i \) given set \( M \) of simultaneous transmitters, and \( \theta \) is the threshold for the successful decoding of the received signal, which depends on the modulation scheme, target bit-error-rate, and the number of bits in the packet, i.e., the transmission rate. Once the channel statistics are known, the packet reception probabilities can be readily computed. In Appendix A, Eq. (1) is evaluated in a Rayleigh fading environment. Of course, Eq. (1) is an approximation since it does treat interference as white Gaussian noise, however, it is used widely and represents a compromise between accuracy and cross-layer modeling [18].

We adopted the notion of stability used in [19] where the stability of a queue is equivalent to the existence of a proper limiting distribution. In other words, a queue is said to be stable if

\[
\lim_{n \to \infty} \Pr[Q_i(n) < x] = F(x) \quad \text{and} \quad \lim_{x \to \infty} F(x) = 1.
\]
If a weaker condition holds, namely,

$$\lim_{x \to \infty} \liminf_{n \to \infty} \Pr\{Q_i(n) < x\} = 1$$

the queue is said to be substable or bounded in probability. Otherwise, the queue is unstable. A stable queue is necessarily substable, but a substable queue is stable if the distribution tends to a limit. If $Q_i(n)$ is an aperiodic and irreducible Markov chain defined on a countable space, which is the case considered in this paper, substability is equivalent to the stability and it can be understood as the recurrence of the chain. Both the positive and null recurrence imply stability because a limiting distribution exists for both cases although the latter may be degenerate. Loynes’ theorem, as it relates to stability, plays a central role in our approach [20]. It states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, the queue is stable. If the average arrival rate is greater than the average service rate, the queue is unstable and the value of $Q_i(n)$ approaches infinity almost surely. If they are equal, the queue can be either stable or substable but in our case the distinction is irrelevant, as mentioned earlier. Finally, the stability region of the system is defined as the pair of arrival rates $(\lambda_1, \lambda_2)$ for which the queues at both $s_1$ and $s_2$ are stable by considering all feasible multi-access probability $p$.

III. Stability in the Presence of Sensing Errors

A. Background on the Spectrum Sensing

Spectrum sensing is the task of obtaining awareness about the existence of the primary user’s transmission over the shared channel. It is mandatory by IEEE 802.22 that the secondary user must perform sensing before making any transmission decision over the licensed spectrum [21]. The goal of this section is to point out some fundamental aspects of the spectrum sensing. Please refer [3], [11], and references therein for more details on various spectrum sensing techniques. Among many others, the energy detector is the most popular because of its low computational and implementation complexity. In addition, it is more generic as receivers do not need any a priori knowledge on the primary user’s signal waveform [11]. The output of the energy detector, which is the sum of the sampled received signal power, is compared to a certain threshold $\tau$ to decide the existence of the primary user’s signal as illustrated in Fig. 1. The performance of a detector can be specified in terms of the probability of miss $\epsilon_m$ and the probability of false alarm $\epsilon_f$. Let $\bar{\epsilon}_m = 1 - \epsilon_m$ and $\bar{\epsilon}_f = 1 - \epsilon_f$, which are the probabilities of detection and correct
rejection, respectively. With phase-shift keying (PSK) modulation and circular symmetric Gaussian noise modeling, the probability of detection with the energy detector was derived as [10]

\[ \bar{\epsilon}_m = Q\left( \frac{1}{\sqrt{2\gamma} + 1} \left( Q^{-1}(\epsilon_f) - \sqrt{T_s f_s \gamma} \right) \right) \]  

(2)

where \( Q(\cdot) \) is the Q-function, i.e., the tail probability of the standard Gaussian distribution, \( \gamma \) is the received SNR of the signal transmitted from \( s_1 \) at the detector at \( s_2 \), \( T_s \) denotes the sensing time, and \( f_s \) is the sampling frequency. In Fig. 3, the receiver operating characteristic (ROC) curve, i.e., (2), is plotted for different received SNR values. In general, \( \epsilon_m \) and \( \epsilon_f \) are in a trade-off relationship, as observed in the figure, since one can always be made arbitrarily small at the expense of the other [22]. Specifically, any point on a given curve can be attained by controlling the threshold \( \tau \) for the detection. It is assumed throughout the paper that \( \bar{\epsilon}_m > \epsilon_f \), which simply indicates that the equipped detector performs better than the pure random guessing\(^2\) whose ROC curve is the diagonal line connecting (0, 0) and (1, 1) in the

\(^2\)The random guessing, which completely ignores the observation, can be done by running coin tossing, and each point on the diagonal line can be achieved by altering the probability of head.
B. Main Result on the Stability Region

In this section, we describe the stability region of the cognitive access system in the presence of sensing errors. This enables us to judge the stability of the system at any given input rate vector. As noted earlier, the queues in the system are interacting, which makes the analysis challenging. The proof of the main results described in this section is presented in Section IV which can be outlined as follows: we first obtained the stability region for given multi-access probability $p$ using the stochastic dominance technique [13]. Since an input rate vector that is outside of the stability region at a certain multi-access probability may be stably supported by another feasible multi-access probability, determination of the closure of the stability region is necessary and important. Thus, we take the closure of the stability region over all feasible values of $p$, which is what is described in this section.

Define $\Delta_i = q_{ii} - q_{i\{1,2\}}$, $i \in \{1, 2\}$, which is the difference between the success probabilities when $s_i$ transmits alone and when it transmits along with $s_j$ ($j \neq i$). The quantity $\Delta_i$ is strictly positive since interference only reduces the probability of success. Let us further define

$$\eta \triangleq q_{i\{i\}} q_{i\{1\}} q_{i\{1,2\}} + q_{i\{2\}} q_{i\{1\}} - q_{i\{1\}} q_{i\{2\}}$$

which can be viewed as an indicator of the degree of the capture effect. In the case of the collision channel, for instance, it is given by $q_{i\{i\}} = 1$ and $q_{i\{1,2\}} = 0$, $\forall i \in \{1, 2\}$ and, thus, $\eta = -1$. On the contrary, in the case of the perfect orthogonal channel with $q_{i\{i\}} = q_{i\{1,2\}} = 1$, $\forall i \in \{1, 2\}$, we have $\eta = 1$.

Described below is our main finding, which is a sufficient and necessary condition for the stability of the considered cognitive access system.

- Case $A$: If $\eta \geq 0$, the stability region of the system is given by the union of the following subregions:

$$\mathcal{R}_1^A = \left \{ (\lambda_1, \lambda_2) : \lambda_2 \leq \frac{\Delta_2}{q_{i\{1,2\}}} \lambda_1, 0 \leq \lambda_1 \leq I_1^A \right \}$$

$$\mathcal{R}_2^A = \left \{ (\lambda_1, \lambda_2) : \lambda_2 \leq \frac{q_{i\{1\}} q_{i\{1,2\}}}{\Delta_1} (q_{i\{1\}} - \lambda_1), I_1^A < \lambda_1 \leq q_{i\{1\}} \right \}$$

where $I_1^A = q_{i\{1,2\}}$. The region is depicted in Fig. 4, which is a convex polygon. The entire boundary of the region can be achieved with multi-access probability $p^* = 1$. Note that the stability region does not depend on sensing error rates.
Fig. 4. Illustration of the stability region for Case A (parameter setting: $q_{1|1} = q_{2|2} = 0.9$, $q_{1|1,2} = q_{2|1,2} = 0.6$ with any positive values of $\epsilon_m$ and $\epsilon_f$)

- Case B: If $-q_{2|2} \epsilon_f \Delta_1 \leq \eta < 0$, the stability region is given by the union of the following subregions:

$$\mathcal{R}_1^B = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq q_{2|2} - \frac{\Delta_2}{q_{1|1,2}} \lambda_1, 0 \leq \lambda_1 \leq I_1^B \right\},$$

$$\mathcal{R}_2^B = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq \left( -\eta' - \sqrt{-\eta' q_{2|2} \epsilon_f \lambda_1} \right)^2 + \frac{q_{2|1,2} (q_{1|1} - \lambda_1)}{q_{2|1,2} \epsilon_f \Delta_1}, I_1^B < \lambda_1 \leq I_2^B \right\},$$

$$\mathcal{R}_3^B = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq \frac{q_{2|1,2}}{\Delta_1} (q_{1|1} - \lambda_1), I_2^B < \lambda_1 \leq q_{1|1} \right\}.$$

where $\eta' = \epsilon_m \eta - q_{1|1,2} q_{2|2} \epsilon_f$, $I_1^B = \frac{q_{1|1,2} q_{2|2} \epsilon_f}{-\eta'}$, and $I_2^B = \frac{-\eta'}{q_{2|2} \epsilon_f}$. Note that $\eta < 0$ implies $\eta' < 0$ but the converse is not true. The boundary of the subregion $\mathcal{R}_1^B$ is achieved with $p^* = 1$, that of $\mathcal{R}_2^B$ is achieved with

$$p^* = \frac{q_{1|1} - \epsilon_m \Delta_1 - \sqrt{-\eta' \lambda_1}}{\epsilon_m \Delta_1} \quad (3)$$

and that of $\mathcal{R}_3^B$ is achieved with $p^*$ given by Eq. (3) evaluated at $\lambda_1 = I_2^B$. The region is non-convex as illustrated in Fig. 5. This follows from the non-convexity of the region described by $\mathcal{R}_2^B$. Also,
the slope of the boundary of the subregion $\mathcal{R}_1^B$ is steeper than that of $\mathcal{R}_3^B$ for the considered case.

- **Case C**: If $\eta < -q_2[2] \epsilon_f \Delta_1$, the stability region is given by the union of the following subregions:

  \[ R_1^C = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq q_2[2] - \frac{\Delta_2}{q_1[1]} \lambda_1, 0 \leq \lambda_1 \leq I_1^C \right\}, \]

  \[ R_2^C = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq \frac{\sqrt{-\eta} - \sqrt{q_2[2] \epsilon_f \lambda_1}}{\epsilon_m \Delta_1} + \frac{q_2[1,2] (q_1[1] - \lambda_1)}{\Delta_1}, I_1^C < \lambda_1 \leq I_2^C \right\}, \]

  \[ R_3^C = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq q_2[2] \epsilon_f - \frac{q_2[1,2] \epsilon_m}{q_1[1]} \lambda_1, I_2^C < \lambda_1 \leq I_3^C \right\}, \]

  \[ R_4^C = \left\{ (\lambda_1, \lambda_2) : \lambda_2 \leq \frac{q_2[1,2]}{\Delta_1} (q_1[1] - \lambda_1), I_3^C < \lambda_1 \leq q_1[1] \right\} \]

where $I_1^C = \frac{q_2[1,2] q_2[2] \epsilon_f}{-\eta}$, $I_2^C = \frac{q_2[2] \epsilon_f (q_1[1] - \epsilon_m \Delta_1)^2}{-\eta}$, and $I_3^C = q_1[1] - \epsilon_m \Delta_1$. As in Case B, the boundary of subregion $\mathcal{R}_1^C$ is achieved with $p^* = 1$, and that of $\mathcal{R}_2^C$ is achieved with $p^*$ given in Eq. (3) which diminishes from one to zero as $\lambda_1$ increases from $I_1^C$ to $I_2^C$. For the boundary of subregions $\mathcal{R}_3^C$ and $\mathcal{R}_4^C$, it is given by $p^* = 0$. The entire region is non-convex as in Case B.

**Remark 3.1**: Consider the case with perfect sensing whose operating point is the upper left corner on
the ROC space as shown in Fig. 3. By substituting $\epsilon_f = \epsilon_m = 0$ into the descriptions of the stability region given above, we find the stability region for the case with perfect sensing, which reconfirms the previous result obtained in [8]. For comparison’s sake, it is also depicted in Fig. 4 to 6 along with the case of imperfect sensing. Most importantly, it is observed from Fig. 4 that the stability region is not affected by the sensing errors when $\eta \geq 0$. This is because the boundary achieving multi-access probability is equally given by $p^* = 1$ regardless of the values of sensing error rates. In other words, when relatively strong capture effect presents which is indicated by $\eta$, it is beneficial to let the secondary node access the channel persistently and aggressively regardless of the sensing outcome, whenever it has non-empty queue. In contrast, from Fig. 5 and 6, it is observed that when $\eta < 0$, the system suffers from the sensing errors. The difference between the regions, therefore, can be understood as the loss due to the imperfect sensing. In Appendix A, the parameter $\eta$ is evaluated by incorporating physical layer parameters such as distances between nodes and their transmit power.
C. Controlling the Operating Point of the Detector

In this section, the problem of controlling the operating point of the sensing device is studied. Clearly, Case $A$ in Section III-B, i.e., when $\eta \geq 0$, is not of our concern because the stability region is not affected by sensing errors. On the other hand, when $\eta < 0$, which includes both Case $B$ and Case $C$, there arises a need for optimally choosing the operating point of the sensing device. However, it is not easy to find certain decisive rules on the control from the descriptions on the stability region, which is complicated with sensing error rates and packet reception probabilities. Moreover, at $\epsilon_f^* = -\eta/q_{2|2}\Delta_1$, the stability region experiences a transition from Case $B$ to Case $C$, and no simple relationship exists between them such as one becomes a subset of the other. Instead, we summarize some general aspects observed by changing the operating point of the sensing device. The results are demonstrated particularly for the energy detectors introduced in Section III-A, but the statements made in this section hold for any rational detector satisfying the following mild conditions: i) its ROC curve connects the points $(0, 0)$ and $(1, 1)$ in Fig. 3, ii) the superiority over the random guessing, i.e., $\bar{\epsilon}_m > \epsilon_f$, and iii) the monotonicity in

![Diagram of stability region transition](image)
the trade-off between $\epsilon_m$ and $\epsilon_f$.

We first observe that increasing $\epsilon_f$ over $\epsilon_f^\star$ only reduces the stability region as shown in Fig. 7. Thus, it is necessary to lower $\epsilon_f$ up to $\epsilon_f^\star$, which directly increases the probability of correct rejection and, thereby, improves the utilization of idle slots, although, at the same time, it risks the success of the primary user by increasing the probability of miss. We next observe that when $\epsilon_f$ is further lowered below $\epsilon_f^\star$, the inclusion relation does not hold anymore and, as illustrated in Fig. 8, each operating point results in a different shape that is not a proper subset of the others at different operating points. Our key observations made in this section are summarized as follows.

- If $\eta \geq 0$, controlling the operating point of detector is not needed in terms of the achieved stability region.
- If $\eta < 0$, lowering $\epsilon_f$ up to $\epsilon_f^\star = -\eta/q_2(2)\Delta_1$ gives monotonic increase in the stability region. Further lowering $\epsilon_f$ below $\epsilon_f^\star$ does not guarantee such monotonic increase.

We finally consider the case when the accuracy of the sensing device itself is improved from the random guessing to the ideal sensing. In Fig. 9, It is obvious that the stability region becomes larger as the sensing
operating point is moved from \((0.5, 0.5)\) to \((0, 1)\) on the ROC plane with the step size of 0.1. Note that in the case of the ideal sensing, the stability region becomes a right triangle as shown in the figure which agrees with the previous result obtained in [8]. On the other hand, the stability region for the case with random guessing can be obtained by substituting \(\bar{\epsilon}_m = \epsilon_f\) into the descriptions in Section III-B and, interestingly, it turns out to be identical with that obtained for the two-node random access system [14]. This indicates that if the cognitive access system is based on the sensing information that is nothing but randomly guessed, its performance is not better than that of the random access system.

IV. Analysis Using the Stochastic Dominance Technique

In this section, we provide details on the derivation of our main results presented in the previous section. In the considered protocol, primary user \(s_1\) transmits a packet whenever its queue is non-empty, independent of the actions made by the secondary user \(s_2\). Secondary user \(s_2\), on the other hand, makes use of the ability to sense before transmitting. If \(s_1\) is observed to be idle, \(s_2\) transmits with probability 1 given that its queue is non-empty. Otherwise, if \(s_1\) is observed to be active, \(s_2\) transmits with probability
p. The probability that $s_1$ is sensed to be idle is $\bar{\epsilon}_f$ when $s_1$ is indeed idle and $\epsilon_m$ when $s_1$ is actually active. Similarly, the probability that $s_1$ is sensed to be active is $\bar{\epsilon}_m$ when it is indeed active and $\epsilon_f$ when it is actually idle. Taking these into account, the average service rates of the users can be written as

$$\mu_1 = q_{1|\{1\}} \left( \Pr[Q_2 = 0] + \Pr[Q_2 \neq 0] \bar{\epsilon}_m (1 - p) \right) + q_{1|\{1,2\}} \Pr[Q_2 \neq 0] (\epsilon_m + \bar{\epsilon}_m p),$$

$$\mu_2 = q_{2|\{2\}} \Pr[Q_1 = 0] (\epsilon_f + \epsilon_f p) + q_{2|\{1,2\}} \Pr[Q_1 \neq 0] (\epsilon_m + \bar{\epsilon}_m p)$$

where $Q_i$ denotes the number of packets in the queue at $s_i$. **If the queue is stable, $Q_i$ denotes the steady-state number of packets in the queue, whereas if the queue is unstable, it is an asymptotic number of packets in the queue that might approach infinity as time goes infinity.**

In the previous work, it is often required that the probability of detection must be above a certain value as a protection for the primary user. However, this is based on the assumption that the secondary user is always backlogged and, therefore, the occurrence of the missed detection directly results in the interference to the primary user. In the practical system with bursty packet arrivals, however, it is unclear how users interfere with each other since they transmit only when having non-empty queues, and this is the reason why we focus on the queueing stability of the system.

Note that the rates of the individual departure processes described above cannot be computed directly, as they are interdependent, without knowing the stationary probability of the joint queue length process. We bypass this difficulty by using the stochastic dominance technique, which was introduced in [13] to deal with such an interacting system in the context of random access stability and has been used by many others in different contexts [9], [14], [23]–[25]. The analysis technique used in this work for the cognitive radio network is exactly the same as for the slotted ALOHA in [13], although the characteristics of primary and secondary users are a bit different from those of the random access participants. The essence of the stochastic dominance technique is to decouple the interaction between queues via the construction of a hypothetical system; this hypothetical system operates as follows: i) the packet arrivals at each node occur at exactly the same instants as in the original system, ii) the coin toss that determines the multi-access by the secondary node has exactly the same outcome in both systems, iii) however, one of the nodes in the system continues to transmit dummy packets even when its packet queue is empty. Sending dummy packets is only aimed to cause constant interference to the other node and does not contribute to throughput if the transmission is successful.
A. First Dominant System: Secondary User Transmits Dummy Packets

Construct a hypothetical system which is identical to the original system except that the secondary user $s_2$ transmits dummy packets when it decides to transmit but when its packet queue is empty. Thus, $s_2$ transmits with probability 1 if $s_1$ is sensed to be idle and with probability $p$ if $s_1$ is sensed to be active, regardless of the emptiness of its queue. Hence, from (4), the average service rate of $s_1$ is obtained as

$$\mu_1 = q_{1|\{1\}}\bar{\epsilon}_m (1 - p) + q_{1|\{1,2\}} (\epsilon_m + \bar{\epsilon}_m p)$$

which can be rewritten as

$$\mu_1 = q_{1|\{1\}} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p.$$  

(6)

By Loynes’ Theorem, the queue at $s_1$ is stable if $\lambda_1 \leq \mu_1$, and the content size follows a discrete-time $M/M/1$ model with the arrival rate $\lambda_1$ and the service rate $\mu_1$. For a stable input rate $\lambda_1$, the queue at $s_1$ empties out with probability given by

$$\Pr[Q_1 = 0] = 1 - \frac{\lambda_1}{\mu_1} = 1 - \frac{\lambda_1}{q_{1|\{1\}} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p}.$$  

(7)

By substituting (7) into (5), the average service rate of the queue at $s_2$ is obtained as

$$\mu_2 = q_{2|\{2\}} (\bar{\epsilon}_f + \epsilon_f p) + \frac{q_{2|\{1,2\}} \epsilon_m - q_{2|\{2\}} \bar{\epsilon}_f + (q_{2|\{1,2\}} \bar{\epsilon}_m - q_{2|\{2\}} \epsilon_f) p}{q_{1|\{1\}} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p} \lambda_1$$

(8)

and the queue at $s_2$ is stable if $\lambda_2 \leq \mu_2$. Consequently, for a given multi-access probability $p$, stable input rate pairs $(\lambda_1, \lambda_2)$ are those componentwise less than $(\mu_1, \mu_2)$. Note that in this dominant system in which $s_2$ transmits dummy packets, the behavior of $s_1$ is unaffected by the rate of packet arrivals at $s_2$, i.e., $\lambda_2$, and thus the stability of the queue at $s_1$ is independent of $\lambda_2$. In Fig. 10, we illustrate the obtained stability region at given $p$, where the stability region of the second dominant system is obtained in the subsequent section.

What is important here is that the stability region obtained using the stochastic dominance technique is not merely an inner bound of the stability region of the original system but indeed coincides with that of the original system, i.e., it is sufficient and necessary condition for the stability of the original system. Let us first begin with establishing the sufficient condition for the stability of the original system. It is obvious that sample-pathwise the queue sizes in this dominant system will never be smaller than their counterparts in the original system, provided the queues start with identical
initial conditions. Thus, the stability condition obtained for the dominant system is a sufficient condition for the stability of the original system. It turns out, however, that it is indeed sufficient and necessary. In the first dominant system, let us choose $\lambda_1$ such that the queue at $s_1$ is stable and $\lambda_2$ such that the queue at $s_2$ is unstable, e.g., point $B$ in Fig. 10. Since the queues in the dominant system contain at least as many packets as in the original system, a queue that is stable in the dominant system is also stable in the original system and, hence, the queue at $s_1$ is stable in the original system as well. Thus, here we are concerned only about the instability of the queue at $s_2$. Any sample path for the queue at $s_1$ is independent of the chosen $\lambda_2$, since $s_2$ makes a decision on its transmission in each time slot regardless of the emptiness of its own queue. As noted in Section II, the stability of a queue is equivalent to the positive recurrence of the Markov chain that represents the evolution of the queue size over time. Since the queue at $s_2$ is unstable, there exist sample paths of the queues, denoted by $sp_1$ and $sp_2$, such that the queue at $s_2$ empties at most a finite number of times before growing without bound. Consider the subpath, denoted by $sp'_i$ of $sp_i$ ($i = 1, 2$) starting from the slot in which the final emptying of the queue at $s_2$ occurs. After
this slot, $s_2$ generates no more dummy packets since it always has packets to send and, hence, it behaves exactly as it would in the original system. In other words, the queue at $s_2$ is never empty along $sp'_2$, $s_2$ in this dominant system does not get any chance to transmit dummy packets, and the behavior of the original system cannot be any different than that of the dominant system. While $sp'_2$ is chosen so that it starts with an empty queue, $sp'_1$ might start with a non-empty finite queue. In that case, $sp'_1$ can be modified to become $sp''_1$ such that packets queued at the beginning of $sp'_1$ are instead assigned as new arrivals in later slots that previously had no packet arrivals; any packets left over from that queue after such assignments are discarded. The queue lengths along $sp''_1$ are adjusted accordingly and are less than or equal to those in $sp'_1$, but the packet arrival rate and transmission behavior for $s_1$ is the same with $sp'_1$ and $sp''_1$. Thus, the sample paths $sp''_1$ and $sp'_2$ exist in both the original and dominant systems, beginning with the empty state on each sample path. Therefore, a value of $\lambda_2$ leading to instability of the queue at $s_2$ in the dominant system will also lead to instability in the original system. This gives us the necessity of the stability condition for the original system. The argument for the other portion of the boundary of the stability region obtained for the hypothetical system in which $s_1$ transmits dummy packets can be made in parallel. The simulation result presented in Section V confirms that the stability region obtained using the stochastic dominance technique is indeed the stability region of the original system.

We now take the closure of the stability region over the multi-access probability $p$. This can be equivalently done by solving the following boundary optimization problem in which we maximize $\mu_2$ over $p$ for a given value of $\lambda_1$ while guaranteeing the stability of the queue at $s_1$, that is

$$\max_p \mu_2 = q_{2|\{2\}} (\bar{\epsilon}_f + \epsilon_f p) + \frac{q_{2|\{1,2\}} \bar{\epsilon}_m - q_{2|\{2\}} \bar{\epsilon}_f + \left( q_{2|\{1,2\}} \bar{\epsilon}_m - q_{2|\{2\}} \epsilon_f \right) p}{q_{1|\{1\}} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p} \lambda_1 \quad (9)$$

subject to

$$0 \leq \lambda_1 \leq q_{1|\{1\}} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p \quad (10)$$

$$0 \leq p \leq 1. \quad (11)$$

To maximize $\mu_2$ over $p$, we need to understand their relationship. Differentiating $\mu_2$ with respect to $p$ gives

$$\frac{\partial \mu_2}{\partial p} = q_{2|\{2\}} \epsilon_f + \frac{\eta' \lambda_1}{\left( q_{1|\{1\}} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p \right)^2}$$

where $\eta'$ was defined as $\eta' = \bar{\epsilon}_m \eta - q_{1|\{1,2\}} q_{2|\{2\}} \epsilon_f$. When $\eta \geq 0$, which is equivalent to the case when
\[ \eta' \geq -q_{1[1,2]}q_{2[2]}\epsilon_f, \] we observe that

\[
\frac{\partial \mu_2}{\partial p} \geq q_{2[2]}\epsilon_f - \frac{q_{1[1,2]}q_{2[2]}\epsilon_f \lambda_1}{(q_{1[1]} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p)^2} \\
\geq q_{2[2]}\epsilon_f \left( 1 - \frac{q_{1[1,2]}}{q_{1[1]} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p} \right) \\
\geq 0
\]

where the last inequality follows from

\[
q_{1[1]} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p \geq q_{1[1]} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 \\
= q_{1[1]} - \Delta_1 \\
= q_{1[1,2]}.
\]

Thus, if \( \eta \geq 0 \), \( \mu_2 \) is a non-decreasing function of \( p \). Note, however, that having \( \eta < 0 \) does not necessarily mean that \( \mu_2 \) is a non-increasing function of \( p \). By differentiating \( \mu_2 \) once again, we have

\[
\frac{\partial^2 \mu_2}{\partial p^2} = \frac{2\bar{\epsilon}_m \Delta_1 \eta' \lambda_1}{(q_{1[1]} - \epsilon_m \Delta_1 - \bar{\epsilon}_m \Delta_1 p)^3}.
\]

Since the denominator is strictly positive, if \( \eta' \geq 0 \), \( \mu_2 \) is convex with respect to \( p \). Otherwise, it is concave with respect to \( p \). These properties of \( \mu_2 \) are described in Fig. 11.
1) The case when $\eta \geq 0$: In this case, $\mu_2$ is a non-decreasing function of $p$. Thus, the maximizing $p^*$ is the largest value satisfying both constraints in Eqs. (10) and (11), that is

$$p^* = \min \left[ 1, \frac{q_{1\{1\}} - \epsilon_m \Delta_1 - \lambda_1}{\epsilon_m \Delta_1} \right].$$

Note that the role of Eq. (10) is to impose an upper limit on $p^*$ so that the stability of $s_1$ is guaranteed. For $0 \leq \lambda_1 \leq q_{1\{1,2\}}$, it is given by $p^* = 1$, and the corresponding maximum function value is obtained as

$$\mu^*_{2,\text{line}_1} = q_{2\{2\}} - \frac{\Delta_2}{q_{1\{1,2\}}} \lambda_1. \quad (12)$$

For $q_{1\{1,2\}} < \lambda_1 \leq q_{1\{1\}} - \epsilon_m \Delta_1$, it is given by $p^* = (q_{1\{1\}} - \epsilon_m \Delta_1 - \lambda_1)/\epsilon_m \Delta_1$, and the corresponding maximum function value is obtained as

$$\mu^*_{2,\text{line}_2} = \frac{q_{2\{1,2\}}}{\Delta_1} (q_{1\{1\}} - \lambda_1). \quad (13)$$

Note that if $\lambda_1 > q_{1\{1\}} - \epsilon_m \Delta_1$, the constraint in Eq. (10) cannot be met with any feasible $p \in [0, 1]$ and, thus, $\mu_2$ is not defined.

2) The case when $-q_{2\{2\}} \epsilon_f \Delta_1 \leq \eta < 0$: In this case, $\mu_2$ is concave with respect to $p$ and, thus, equating the first derivative to zero gives the maximizing $p^*$ as

$$p^* = \frac{q_{1\{1\}} - \epsilon_m \Delta_1 - \sqrt{-\eta' \lambda_1}}{\epsilon_m \Delta_1 \sqrt{q_{2\{2\}} \epsilon_f}} \quad (14)$$

and the corresponding maximum function value is obtained as

$$\mu^*_{2,\text{curve}} = \frac{(q_{2\{2\}} \epsilon_f - q_{2\{1,2\}} \epsilon_m) \lambda_1 - 2 \sqrt{-q_{2\{2\}} \epsilon_f \eta' \lambda_1 - \eta' + q_{1\{1\}} q_{2\{1,2\}} \epsilon_m}}{\epsilon_m \Delta_1}$$

which can be rearranged to

$$\mu^*_{2,\text{curve}} = \frac{(\sqrt{-\eta'} - \sqrt{q_{2\{2\}} \epsilon_f \lambda_1})^2}{\epsilon_m \Delta_1} + \frac{q_{2\{1,2\}} (q_{1\{1\}} - \lambda_1)}{\Delta_1}.$$ 

Note that $\mu^*_{2,\text{curve}}$ is feasible when both constraints in Eqs. (10) and (11) are satisfied. For used $p^*$, Eq. (10) becomes

$$\lambda_1 \leq \frac{-\eta'}{q_{2\{2\}} \epsilon_f} \quad (15)$$
and Eq. (11) becomes
\[ \frac{q_1^{2} q_{2\{2\}} \epsilon_f}{-\eta'} \leq \lambda_1 \leq \frac{q_{2\{2\}} \epsilon_f (q_{1\{1\}} - \epsilon_m \Delta_1)^2}{-\eta'} \] (16)

which is obtained by rearranging Eq. (14) and substituting the extreme values of \( p \). For the considered case when \(- q_{2\{2\}} \epsilon_f \Delta_1 \leq \eta < 0\), the intersection of the ranges of values of \( \lambda_1 \) determined by Eqs. (15) and (16) is given by
\[ \frac{q_1^{2} q_{2\{2\}} \epsilon_f}{-\eta'} \leq \lambda_1 \leq -\frac{\eta'}{q_{2\{2\}} \epsilon_f}. \] (17)

On the other hand, if \( \lambda_1 \) lies on the left-hand side (LHS) of the range of Eq. (17), we observe that
\[ \frac{\partial \mu_2}{\partial p} \geq q_{2\{2\}} \epsilon_f \left( 1 - \left( \frac{q_{1\{1,2\}} \epsilon_f}{q_{1\{1\}} - \epsilon_m \Delta_1 - \epsilon_m \Delta_1 p} \right)^2 \right) \geq 0 \]
where we used the facts that \( \eta' \) is negative for the considered case and \( q_{1\{1\}} - \epsilon_m \Delta_1 - \epsilon_m \Delta_1 p \geq q_{1\{1,2\}} \) as observed in the previous case. Since \( \mu_2 \) is a non-decreasing function of \( p \), \( p^* = 1 \) and the maximum function value is given by \( \mu_{2,\text{line1}}^* \) in Eq. (12). Note that the constraint in Eq. (10) is automatically satisfied when \( \lambda_1 \) is on the LHS of the range of Eq. (17).

Next consider the case when \( \lambda_1 \) lies on the right-hand side (RHS) of the range of Eq. (17). This is the case that, if \( p^* \) is set according to Eq. (14), the stability of \( s_1 \) is lost. For the stability of \( s_1 \), it is required that the multi-access probability \( p \) is bounded above as
\[ p \leq \frac{q_{1\{1\}} - \epsilon_m \Delta_1 - \lambda_1}{\epsilon_m \Delta_1} < \frac{q_{1\{1\}} - \epsilon_m \Delta_1 + \frac{\eta'}{q_{2\{2\}} \epsilon_f}}{\epsilon_m \Delta_1}. \]

For \( p \) satisfying the above inequality, we observe that
\[ \frac{\partial \mu_2}{\partial p} > q_{2\{2\}} \epsilon_f \left( 1 - \left( \frac{-\eta'}{q_{2\{2\}} \epsilon_f (q_{1\{1\}} - \epsilon_m \Delta_1 - \epsilon_m \Delta_1 p)} \right)^2 \right) > 0. \]
In other words, \( \mu_2 \) is an increasing function of \( p \) and, hence, we have
\[ p^* = \frac{q_{1\{1\}} - \epsilon_m \Delta_1 - \lambda_1}{\epsilon_m \Delta_1} \]
for \( \lambda_1 \) on the RHS of the range of Eq. (17). The corresponding maximum function value is given by \( \mu_{2,\text{line2}}^* \) in Eq. (13). Again, if \( \lambda_1 > q_{1\{1\}} - \epsilon_m \Delta_1 \), the constraint in Eq. (10) cannot be met with any feasible \( p \in [0, 1] \) and, thus, \( \mu_2 \) is not defined.
3) The case when \( \eta < -q_{2|\{2\}} \epsilon_f \Delta_1 \): In this case, \( \mu_2 \) is still concave with respect to \( p \), but the range of \( \mu_{2,\text{curve}} \), which was the intersection of the ranges of values of \( \lambda_1 \) determined by Eqs. (15) and (16), would be identical with the range specified by (16). Again, for \( \lambda_1 \) on the LHS of the range of Eq. (16), \( \mu_2 \) is a non-decreasing function of \( p \), and the maximum function value is given by \( \mu_{2,\text{line1}}^* \) as in the previous case. On the other hand, if \( \lambda_1 \) lies on the RHS of Eq. (16), we observe that

\[
\frac{\partial \mu_2}{\partial p} < q_{2|\{2\}} \epsilon_f \left( 1 - \left( \frac{q_{1|\{1\}} - \epsilon_m \Delta_1}{q_{1|\{1\}} - \epsilon_m \Delta_1 - \epsilon_m \Delta_1 p} \right)^2 \right) < 0.
\]

Therefore, \( \mu_2 \) is a decreasing function of \( p \) and, hence, by substituting \( p^* = 0 \) into (9), we have

\[
\mu_{2,\text{line3}}^* = q_{2|\{2\}} \epsilon_f - \frac{q_{2|\{2\}} \epsilon f - q_{2|\{1,2\}} \epsilon m}{q_{1|\{1\}} - \epsilon_m \Delta_1} \lambda_1.
\]

For \( \lambda_1 > q_{1|\{1\}} - \epsilon_m \Delta_1 \), \( \mu_2 \) is not defined.

B. Second Dominant System: Primary User Transmits Dummy Packets

By reversing the roles of the two users in the previous dominant system, we can construct another parallel dominant system in which the primary user \( s_1 \) is now transmitting dummy packets, instead of the secondary node \( s_2 \), when its packet queue is empty. Since \( s_1 \) transmits with probability 1 in this dominant system, the average service rate of \( s_2 \) in (5) becomes

\[
\mu_2 = q_{2|\{1,2\}} (\epsilon_m + \epsilon_m p).
\]

By Loynes’ theorem, the queue at \( s_2 \) is stable if \( \lambda_2 \leq \mu_2 \), and it empties out with probability given by

\[
\Pr[Q_2 = 0] = 1 - \frac{\lambda_2}{q_{2|\{1,2\}} (\epsilon_m + \epsilon_m p)}.
\]

(18)

Substituting (18) into (4) and after some manipulation, the stability condition for the queue at \( s_1 \) is obtained as

\[
\lambda_1 \leq \mu_1 = q_{1|\{1\}} - \frac{\Delta_1}{q_{2|\{1,2\}}} \lambda_2
\]

which is depicted in Fig. 10 for the range of \( \lambda_2 \leq \mu_2 \). Observe that Eq. (19) can be rearranged to

\[
\lambda_2 \leq \frac{q_{2|\{1,2\}}}{\Delta_1} (q_{1|\{1\}} - \lambda_1)
\]

(20)
Fig. 12. Average queue length over $10^6$ slots along the diagonal arrow in Fig. 10, i.e., $\lambda = \lambda_1 = \lambda_2$, with the same parameter setting used for Fig. 10

whose boundary is identical with $\mu^*_2, \text{line}_2$ in Eq. (13) for the range of $\lambda_1 \geq q_{1|\{1\}} - \Delta_1 (\epsilon_m + \bar{\epsilon}_m p)$. Since Eq. (20) does not depend on $p$, there is no need to optimize over $p$, and $p$ only has the effect of changing the range of $\lambda_1$. This together with the descriptions obtained for the first dominant system completes the proof of our main results presented in Section III.

V. Simulation

Here we focus on the validation of the stability result obtained using the stochastic dominance technique, which is illustrated in Fig. 10. The simulation results reconfirm that the stability region in Fig. 10 is indeed the stability region of the original system. Results illustrated from Fig. 4 to Fig. 9 are consequences of the stability region in Fig. 10 by taking the closure over the multi-access probability $p$ by the secondary transmitter.

We first observe the behavior of the average queue sizes as the traffic load increases. For simplicity of exposition, we consider symmetric Bernoulli arrivals, so that $\lambda_i = \lambda$ for all $i$ in $\{1, 2\}$. We simulated the system over $10^6$ slots with the same parameter setting used for Fig. 10. That is, $q_{1|\{1\}} = q_{2|\{2\}} = 0.9$,
Fig. 13. Queue length samplepath at point A in Fig. 10

$q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.3$, $\epsilon_m = \epsilon_f = 0.3$, and $p = 0.5$. The resulting simulated queue averages are shown in Fig. 12. It can be observed that as we cross the boundary of the stability region, which is at $\lambda = 0.3613$ from Eq. (8), the size of the queue at $s_2$ starts growing. As the input rate is further increased such that it exceeds $\lambda = 0.51$, which is obtained from Eq. (6), we observe that the size of the queue at $s_1$ also starts growing.

However, this result is still insufficient because stability is an asymptotic property of a system but the queue length averages were taken over the finite time interval. Although the simulation interval can be further increased, it is nevertheless finite. Thus, the best we can do is to observe the tendency of queue sizes over the course of time. From Fig. 13 to Fig. 15, we illustrate queue length sample-paths at input traffic load at points $A$, $B$, and $C$ depicted in Fig. 10. At point $A$, the queues at both $s_1$ and $s_2$ are expected to be stable, whereas at points $B$ and $C$, the queue at $s_2$ and the queues at both $s_1$ and $s_2$ are expected to be unstable, respectively. From Fig. 13, it can be seen that the queue sizes at both nodes do not increase as time goes on, albeit bursty. From Fig. 14 and Fig. 15, it can be observed that there is an increasing tendency in the queue size at $s_2$ and the queue sizes at both $s_1$ and $s_2$, respectively. This
increasing tendency in the queue size allows us to conjecture the instability of the corresponding queues.

VI. CONCLUDING REMARKS

We studied the effect of imperfect sensing on the stability region of the cognitive access system and observed that when there exists relatively strong capture effect, we can achieve the identical stability region that is achieved with perfect sensing, even with positive sensing error rates. This is remarkable because the spectrum sensing itself becomes unnecessary in terms of the achieved stability region, although other performance measures such as the average queueing delay or energy efficiency may suffer from the occurrence of errors. When it is not the case that we can achieve the identical stability region that is achieved with perfect sensing, the loss due to the imperfect sensing was precisely quantified in terms of the size of the stability region.

The focus of this work was on the exact characterization of the stability region of cognitive radio systems in the presence of sensing errors for small and apparently simple model involving only two source-destination pairs. Extending the approach proposed here to more realistic environment
with multiple sets of source-destination pairs presents serious difficulties of tractability due to the complex interaction between pairs and requires approximations or alternative approaches. In the context of random access stability, the concept of the instability rank was introduced in [27] to obtain the inner bound of the stability region for general $N$ node case. In [28], an approximation based on the mean-field theory for an arbitrarily large number of nodes with identical arrival rates and transmission probabilities were performed. Since, however, [27] and [28] were based on the collision channel model, extending it to the general SINR-based model, as considered in this work, will be much more challenging. An approximate approach for multiple sets of source-destination pairs is a huge topic in its own right that goes beyond the scope of this work, and we leave it as a possibility of our future investigation.

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APPENDIX A

ON CAPTURE FOR A RAYLEIGH FADING ENVIRONMENT

In this section, the capture probabilities are computed for a Rayleigh fading environment, and the criterion on the convexity of the stability region given by $\eta \geq 0$ is verified in terms of the value of physical layer parameters. The use of matched filters was implicitly assumed in using Eq. (1) for decoding the received signal, which basically treats interference as additive white Gaussian noise. Although, techniques such as the successive interference cancellation [29] can improve the accuracy of the capture probability description, comparing different physical layer techniques is outside the scope of our work here.

We begin by describing the SINR of the signal transmitted from $s_i$ at the corresponding destination $d_i$ as

$$\gamma_{i|M} = \frac{P_{rx,ii}}{N + \sum_{j \in M \setminus \{i\}} P_{rx,ji}}$$

where $M$ is the set of nodes transmitting simultaneously, $N$ is the background noise power, and $P_{rx,ij}$ is the received power from node $s_i$ at destination $d_j$ which is modeled by

$$P_{rx,ij} = \psi_{ij}^2 K r_{ij}^{-\nu} P_{tx,i}$$

where $\psi_{ij}$ is a Rayleigh random variable with $E[\psi_{ij}^2] = 1$, $K$ is a constant, $\nu$ is the propagation loss exponent, $r_{ij}$ is the distance between $s_i$ and $d_j$, and $P_{tx,i}$ is the transmitted power by $s_i$. Let $f_{\psi_{ij}^2}$ be the probability density function of the fading random variable $\psi_{ij}^2$, which is exponential with unit mean [18]. Then, the success probability of a transmission by $s_i$ when it transmits alone is computed by

$$q_i|\{i\} = \Pr[\gamma_{i|i} \geq \theta] = \int_0^\infty \Pr\left[\omega \geq \frac{\theta N r_{ii}^\nu}{K P_{tx,i}}\right] f_{\psi_{ii}^2}(\omega) d\omega = \exp\left(-\frac{\theta N r_{ii}^\nu}{K P_{tx,i}}\right).$$

Similarly, the success probability of a transmission by $s_i$ when it transmits along with the other node $s_j$
is given by

\[
q_{i \{i,j\}} = \text{Pr} \left[ \frac{\psi_{ii}^2 K r_{ii}^{-\nu} P_{tx,i}}{N + \psi_{ji}^2 K r_{ji}^{-\nu} P_{tx,j}} \geq \theta \right]
\]

\[
= \int_0^\infty \int_0^\infty \text{Pr} \left[ \omega_i \geq \frac{\theta (N + \omega_j K r_{ji}^{-\nu} P_{tx,j})}{K r_{ii}^{-\nu} P_{tx,i}} \right] f_{\psi_{ii}^2}(\omega_i) d\omega_i f_{\psi_{ji}^2}(\omega_j) d\omega_j
\]

\[
= \int_0^\infty \exp \left( -\frac{\theta (N + \omega_j K r_{ji}^{-\nu} P_{tx,j})}{K r_{ii}^{-\nu} P_{tx,i}} \right) f_{\psi_{ji}^2}(\omega_j) d\omega_j
\]

\[
= \left( 1 + \frac{\theta P_{tx,j}}{P_{tx,i}} \left( \frac{r_{ii}}{r_{ji}} \right)^{\nu} \right)^{-1} \exp \left( -\frac{\theta N r_{ii}^{-\nu}}{K P_{tx,i}} \right)
\]

where \( i, j \in \{1, 2\}, j \neq i \), and \( \psi_{ii} \) and \( \psi_{ji} \) were assumed mutually independent.

From Section III, we know that the convexity of the stability region is determined by the sign of \( \eta \), and it is not difficult to observe that

\[
\eta \geq 0 \iff \frac{\Delta_1}{q_{i\{1\}}} + \frac{\Delta_2}{q_{i\{2\}}} \leq 1.
\]  

(21)

By substituting the obtained packet reception probabilities into the definition of \( \Delta_i \), we obtain

\[
\Delta_i = \frac{\theta P_{tx,j}}{P_{tx,i}} \left( \frac{r_{ii}}{r_{ji}} \right)^{\nu} \left( 1 + \frac{\theta P_{tx,j}}{P_{tx,i}} \left( \frac{r_{ii}}{r_{ji}} \right)^{\nu} \right)^{-1} \exp \left( -\frac{\theta N r_{ii}^{-\nu}}{K P_{tx,i}} \right).
\]  

(22)

Then, by substituting \( \Delta_i, \forall i \in \{1, 2\} \), into (21), we express the criterion in terms of the physical layer parameters as

\[
\eta \geq 0 \iff \frac{\theta P_{tx,2} r_{11}^{\nu}}{P_{tx,1} r_{21}^{\nu}} + \frac{\theta P_{tx,2} r_{12}^{\nu}}{r_{22}^{\nu}} \leq 1
\]

\[
\iff \theta^2 r_{11}^{\nu} r_{22}^{\nu} \leq r_{12}^{\nu} r_{21}^{\nu}
\]

\[
\iff \Upsilon \triangleq \left( \frac{r_{12}}{r_{11}} \frac{r_{21}}{r_{22}} \right)^{\nu/2} \geq \theta.
\]

Note that \( \Upsilon \) is expressed only in terms of the distances between sources and destinations and the propagation loss exponent. Thus, changing the transmission power, for example, does not affect the above comparison. Finally, if \( \Upsilon \geq 0 \), we can achieve identical stability region that is achieved with perfect sensing even with positive sensing error rates.

**REFERENCES**


